**The Essence of Vacuum: The Field Combination Mechanism of Quantum Fluctuations—A Rigorous Mathematical Formulation Based on the ABC Theory**

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**Abstract:**  
Based on the ABC theory proposed by Professor Zhijun Li, this paper presents a novel and mathematically rigorous definition of the essence of vacuum. We posit that vacuum is not nothingness, but rather a background sea composed of all possible combination states of the three fundamental cosmic vortex fields—the electromagnetic field A, the color-charge field B, and the Higgs field C—which have failed to form stable observable particles. The essence of quantum fluctuations is the eternal coherent superposition and topological transition between these metastable states and the stable particle states. By constructing a principal fiber bundle model over the gauge group, we define vacuum as the moduli space of flat connections . The statistical laws of quantum fluctuations are rigorously described by the closed-time-path integral, whose generating functional incorporates the Chern-Simons topological term . The expectation value of the Wilson loop operator characterizes the topological fluctuations of vacuum. Calculations show that the vacuum energy density , consistent in order of magnitude with observational values. This theory provides a unified mathematical framework for understanding the nature of vacuum, dark matter, and the cosmological constant problem.

**Keywords:** Vacuum; Field Combination; ABC Theory; Quantum Fluctuation; Principal Fiber Bundle; Chern-Simons Theory; Wilson Loop; Dark Matter; Cosmological Constant

1. **Introduction: The Enigma of Vacuum and the ABC Theoretical Framework**

Quantum field theory reveals that vacuum is not empty but filled with intense quantum fluctuations. However, the traditional image of “virtual particles” is more of a mathematical artifact than a physical explanation. The ABC theory proposed by Li Zhijun provides a novel framework to address this conundrum: the universe is composed of three fundamental vortex fields, whose different combination patterns form all things.

Define the state spaces of the three fundamental fields:  
\* Electromagnetic vortex field A:   
\* Color-charge vortex field B:   
\* Higgs vortex field C:

The total Hilbert space of the system is the tensor product of the three spaces:

From this, the total dimension of the universe can be derived:

This 26-dimensional space contains all possible field combinations, and vacuum is the set of these combination states, minus the excited states that have formed stable particles, leaving the ground or metastable states.

1. **Rigorous Mathematical Definition of Vacuum**

2.1 Algebraic Structure of Field Combinations

Define the creation and annihilation operators for the three fundamental fields:  
\* Electromagnetic field: where denotes polarization  
\* Color-charge field: where denotes the color charge component  
\* Higgs field:

Fourier expansion of the field operators:

The vacuum state is defined as the eigenstate of all annihilation operators:

2.2 Dimensional Reduction and Physical Vacuum

The dimensional reduction process of the physical world can be described by symmetry breaking:  
1. 26D 17D: Merging of electromagnetic fields and breaking symmetry

2. 17D 11D: Merging of Higgs fields and breaking symmetry

In the 11-dimensional space, field combinations form the basis of standard model particles.

2.3 Field Combination Formulation of Dark Matter

Dark matter corresponds to specific field combination states. In the 26-dimensional space, the field operator for genuine dark matter is:

where are operator combinations containing

The equation of motion for the dark matter field:

where is the covariant derivative for dark matter.

1. **Rigorous Mathematical Description of Quantum Fluctuations**

3.1 Closed-Time-Path Integral Formulation

The dynamics of quantum fluctuations are described by the Schwinger-Keldysh closed-time-path integral:

where the time path C goes from to and back.

The generating functional gives all connected correlation functions:

3.2 Topological Field Theory Description: Chern-Simons Theory

The topological properties of vacuum are described by the Chern-Simons action:

where is the connection on the principal bundle, and is the quantized parameter.

The flat connection condition:

Define the moduli space:

where is the gauge transformation group.

3.3 Wilson Loop and Topological Fluctuations

The Wilson loop operator:

Its expectation value characterizes the topological fluctuations of vacuum:

For non-trivial topology, yields topological invariants such as the Jones polynomial.

1. **Vacuum Energy and the Cosmological Constant**

4.1 Zero-Point Energy Calculation

The vacuum energy density originates from the zero-point energy of all field modes:

In the ABC theory, color confinement provides a natural ultraviolet cutoff :

4.2 Precise Calculation and Renormalization

Using dimensional regularization:

where is the renormalization scale.

Calculation results:

Comparison with the observed value requires the introduction of supersymmetry or extra dimension mechanisms for precise matching.

1. **Conclusion and Outlook**

Based on the ABC theory, this paper provides a rigorous mathematical formulation of the essence of vacuum:

1. Algebraic Structure of Vacuum: Vacuum is the set of all combination states of the three fundamental fields, mathematically expressed as
2. Topological Nature of Quantum Fluctuations: Fluctuations are rigorously described by the Chern-Simons action and the Wilson loop
3. Field-Theoretic Formulation of Dark Matter: Dark matter corresponds to specific field combinations
4. Precise Calculation of Vacuum Energy: consistent in order of magnitude with observational values.

Future work will focus on:  
\* Deriving specific scattering amplitudes for field combinations  
\* Calculating the interaction cross-section between dark matter and ordinary matter  
\* Exploring the renormalization mechanism of vacuum energy in extra dimensions

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